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Correlation functions and spin

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The *k*-electron correlation function of a free chaotic electron beam is derived with the spin degree of freedom taken into account. It is shown that this can be expressed with the help of correlation functions for a polarized electron beam of all orders up to *k*, and the degree of spin polarization. The form of the correlation function suggests that if the electron beam is not highly polarized, observing multiparticle correlations should be difficult. The result can also be applied to chaotic photon beams, the degree of spin polarization being replaced by the degree of polarization.

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I. INTRODUCTION

Although the theory of multiparticle correlations in free-electron beams is relatively well developed (see, e.g., Refs. [1–6]), up to now there has not been much attention devoted to electron spin. We feel this is a deficiency because the spin degree of freedom is a significant feature of the electron, and should therefore be taken into account. In this paper we try to reduce this deficiency for one particular case which, as we believe, is the most typical one in electron correlation experiments. That is, we calculate the multielectron correlation function for an electron chaotic beam with an arbitrary degree of spin polarization.

Moreover, the presented theory can be applied not only to chaotic electron (or generally spin-1/2 particle) beams, but also to chaotic photon beams, because the corresponding Hilbert space of possible photon polarization states is two dimensional, precisely as is the Hilbert space of the spin-1/2 states. We will speak about electrons for brevity, but the argumentation and the results can also be applied to the case of photons.

II. CHAOTIC STATE

The reason why we concentrate on the chaotic state is that we believe it is a good approximation of a state produced by the most coherent electron source available nowadays — the field-emission tip. This source is the only candidate for electron correlation experiments at the present time [2], because it offers both high brightness and a relatively monochromatic energy spectrum. The electrons emitted from a field-emission tip originate from a quasiequilibrium state in the metal very close to a thermal state, which is an example of the chaotic state. We do not expect any additional coherence to come into existence during the tunneling process, and therefore suppose that the state also remains chaotic outside the metal.[1]

The chaotic state was first introduced by Glauber for a mode of the electromagnetic field [8] as a state with a maximum entropy for a given occupation number of this mode. We generalize this definition to an electron field with the spin taken into account: the chaotic state is state of maximum entropy for given occupation numbers of the individual modes of the field and for a given spin polarization.

From the condition of maximum entropy it follows that there is no correlation between the coordinate and the spin components of the chaotic state, which allows one to treat the spin degree of freedom separately from the coordinate degrees of freedom. In calculating the correlation functions one can in this way avoid the formalism of spin-dependent electron field operators, and use a relatively simple argumentation based on the probability theory. As will be seen in the following, this enables one to express the correlation functions of a partially spin-polarized chaotic electron beam in terms of the degree of spin polarization and the correlation functions corresponding to a completely polarized beam, the form of which is known [4,6].

For our purpose it is fully sufficient to define the *k*-electron correlation function $O^{(k)}(\vec{r}_1,t_1,\ldots,\vec{r}_k,t_k)$ as the probability of detecting *k* electrons at *k* space-time points $(\vec{r}_1,t_1),(\vec{r}_2,t_2),\ldots,(\vec{r}_k,t_k)$. To obtain a more compact form of the equations, a shortened notation of $O^{(k)}_{12\ldots k}$ will be used instead of $O^{(k)}(\vec{r}_1,t_1,\ldots,\vec{r}_k,t_k)$, each index i standing for one space-time point $(\vec{r}_i,t_i)$. Analogous correlation functions for a spin-polarized beam will be denoted by G instead of O, so, for example, the two-electron correlation function for a polarized beam is $G^{(2)}_{1,2}$. For a more precise definition of correlation functions in terms of electron field operators see, e.g., Ref. [1,6].

III. TWO SPIN-POLARIZED SOURCES

Consider an electron that has been emitted from the source. Its state can be described by the spin density operator $\hat{\rho}_s$ that is represented by a Hermitian matrix of the second order in any orthonormal basis. As $\hat{\rho}_s$ can be diagonalized by a unitary transformation, there exists an orthonormal basis $\{|1\rangle,|2\rangle\}$ in which $\hat{\rho}_s$ has the diagonal form

$$\hat{\rho}_s=\begin{pmatrix}\rho_1 & 0\\ 0 & \rho_2\end{pmatrix}=\rho_1|1\rangle\langle 1|+\rho_2|2\rangle\langle 2|. \tag{1}$$

The states $|1\rangle$ and $|2\rangle$ correspond to spin orientations ‘‘up’’ and ‘‘down,’’ respectively, with respect to some particular

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[1] This might no longer be true for a polarized field-emission source [7], where a correlation between the spin and coordinate can come into existence; however, it can be shown that our results remain valid as long as the energy spectra of the spin components ‘‘up’’ and ‘‘down’’ are close to each other.

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axis $\vec{a}$ in space. In the case of photons, states $|1\rangle$ and $|2\rangle$ would describe two orthogonal polarizations, e.g., two linearly polarized waves with the polarization planes perpendicular to each other or a pair of the left and right circularly polarized waves, depending on the properties of the source.

It is useful to express the probabilities ρ_1 and ρ_2 in terms of the degree of polarization. This quantity is defined as $P = (\rho_1 - \rho_2)/(\rho_1 + \rho_2) = \rho_1 - \rho_2$ (provided that $\rho_1 \geqslant \rho_2$), which yields

$$\rho_1 = \frac{1+P}{2}, \quad \rho_2 = \frac{1-P}{2}. \tag{2}$$

Now the spin state of the ensemble of electrons coming from the source is completely described by the density operator [Eq. (1)]. As can be seen from its form, $\hat{\rho}_s$ corresponds to a situation as if just two types of electrons were emitted from the source: first, electrons polarized up with respect to the axis $\vec{a}$; and second, electrons polarized down with respect to $\vec{a}$. The probabilities that an electron emitted from the source is of the first or second type are ρ_1 or ρ_2, respectively. The fact that the spin density operator $\hat{\rho}_s$ is diagonal means that there is no correlation between the up-spin and down-spin components. At the same time, there is no correlation between the spin and the coordinate because the complete multielectron state is chaotic. This allows the following consideration to be made.

We formally substitute the original electron source S with two independent sources S_{up} and S_{down} that emit electrons polarized up and down with respect to the axis $\vec{a}$, respectively. If this is correct, the S_{up} and S_{down} sources must have the same properties (of course except for the spin) as the original source S. This implies, for example, that they are located at the place of the original source S and have the same energy spectrum. Moreover, the emission intensities of the S_{up} and S_{down} sources must be equal to ρ_1 and ρ_2 times the intensity of the original source S, respectively. In the following, the idea of formally substituting the original source with two polarized ones will be used for a direct derivation of the correlation function for a partially polarized electron beam. To see this idea more clearly, we will consider the simple case $k=2$ first, and then go over to a general k.

IV. TWO-ELECTRON CORRELATION FUNCTION

The two-electron correlation function expresses the probability of the event of detecting two electrons at the space-time points $(\vec{r}_1, t_1)$ and $(\vec{r}_2, t_2)$. This event can occur in one of four ways that are distinguishable in principle because the coordinate and spin operators mutually commute: (1) the spins of both electrons are oriented up with respect to the axis $\vec{a}$; (2) the spins of both electrons are oriented down; (3) the spin of the electron at $(\vec{r}_1, t_1)$ is oriented down, and the spin of the electron at $(\vec{r}_2, t_2)$ is oriented up; and (4) the spin of the electron at $(\vec{r}_1, t_1)$ is oriented up, and the spin of the electron at $(\vec{r}_2, t_2)$ is oriented down. If cases (1) or (2) occur, then according to Sec. III we are dealing with two electrons from the same polarized source. Therefore, the two-electron correlation function is equal to the analogous correlation function $G_{12}^{(2)}$ for polarized electrons. On the other hand, if cases (3) or (4) occur, we are dealing with two electrons from two independent, oppositely polarized sources. The electrons are then completely uncorrelated, and the correlation function is equal to the product of one-electron correlation functions, i.e., $G_1^{(1)} G_2^{(1)}$. As the probability that one electron is polarized up or down is ρ_1 or ρ_2, respectively, the probabilities of cases (1), (2), (3), and (4) are ρ_1^2, ρ_2^2, $\rho_1\rho_2$, and $\rho_1\rho_2$, respectively. The total correlation function $O_{12}^{(2)}$ can be then written as the weighted average of the polarized correlation functions,

$$O_{1,2}^{(2)} = (\rho_1^2 + \rho_2^2) G_{1,2}^{(2)} + 2\rho_1\rho_2 G_1^{(1)} G_2^{(1)}, \tag{3}$$

which is the desired result for $k=2$.

V. *k*-ELECTRON CORRELATION FUNCTION

In the derivation of the k-electron correlation function for a partially polarized chaotic electron beam, we proceed in a completely analogous way. If k electrons at the space-time points $(\vec{r}_1, t_1), \ldots, (\vec{r}_k, t_k)$ should be detected, there are 2^k possibilities how they can be polarized (instead of the four possibilities discussed in Sec. IV). We denote each of them by the sequence $s_1, s_2, \ldots, s_k$, every s_i expressing the spin polarization of the electron at the point $(\vec{r}_i, t_i)$ and having one of two possible values: 1 for spin-up and or 2 for spin-down. The probability $P(s_1, \ldots, s_k)$ that the electrons have polarizations $s_1, \ldots, s_k$ is equal to $\rho_1^{n_1}\rho_2^{n_2}$, where n_1 and n_2 express how many times 1 and 2 appear among the numbers $s_1, \ldots, s_k$, respectively. If $O_{1,\ldots,k}^{(k)}(s_1, \ldots, s_k)$ denotes the k-electron correlation function for this particular spin combination, the total k-electron correlation function can be written as

$$O_{1,\ldots,k}^{(k)} = \sum_{s_1,\ldots,s_k} P(s_1, \ldots, s_k)\, O_{1,\ldots,k}^{(k)}(s_1, \ldots, s_k), \tag{4}$$

the sum being made over all the possibilities $s_1, \ldots, s_k$. Now, if the spin polarizations of the electrons are $s_1, \ldots, s_k$, the situation is the same as if we dealt with two independent sets of electrons—one set of n_1 up-polarized electrons originating from the source S_{up}, and another set of n_2 down-polarized electrons originating from the source S_{down}. The correlation function $O_{1,\ldots,k}^{(k)}(s_1, \ldots, s_k)$ therefore factorizes into a product of two correlation functions for polarized electrons:

$$O_{1,\ldots,k}^{(k)}(s_1, \ldots, s_k) = G^{(n_1)}(\{\vec{r},t\}_{\mathrm{up}})\, G^{(n_2)}(\{\vec{r},t\}_{\mathrm{down}}). \tag{5}$$

Here $\{\vec{r},t\}_{\mathrm{up}}$ and $\{\vec{r},t\}_{\mathrm{down}}$ denote the sets of points at which the electrons are polarized up and down, respectively. Substituting Eq. (5) into Eq. (4) and rearranging the sum, we can write the correlation function $O_{1,\ldots k}^{(k)}$ as follows:

$$\begin{aligned} O_{1,\ldots,k}^{(k)} = {} & (\rho_1^k + \rho_2^k) G_{1,\ldots,k}^{(k)} + (\rho_1^{k-1}\rho_2 + \rho_2^{k-1}\rho_1) \\ & \times (G_{2,\ldots,k}^{(k-1)} G_1^{(1)} + G_{1,3,\ldots,k}^{(k-1)} G_2^{(1)} + \cdots \\ & + G_{1,\ldots,k-1}^{(k-1)} G_k^{(1)}) + (\rho_1^{k-2}\rho_2^2 + \rho_2^{k-2}\rho_1^2) \\ & \times (G_{3,\ldots,k}^{(k-2)} G_{1,2}^{(2)} + G_{2,4,\ldots,k}^{(k-2)} G_{1,3}^{(2)} + \cdots \\ & + G_{1,\ldots,k-2}^{(k-2)} G_{k-1,k}^{(2)}) + \cdots . \end{aligned} \tag{6}$$

To see the structure of such a series better, we write the three- and four-electron correlation functions for illustration:

$$\begin{aligned}O^{(3)}_{1,2,3}=&(\rho_1^3+\rho_2^3)G^{(3)}_{1,2,3}+(\rho_1^2\rho_2+\rho_2^2\rho_1)(G^{(2)}_{1,2}G^{(1)}_3\\&+G^{(2)}_{1,3}G^{(1)}_2+G^{(2)}_{2,3}G^{(1)}_1),\end{aligned}\tag{7}$$

$$\begin{aligned}O^{(4)}_{1,2,3,4}=&(\rho_1^4+\rho_2^4)G^{(4)}_{1,2,3,4}+(\rho_1^3\rho_2+\rho_2^3\rho_1)(G^{(3)}_{1,2,3}G^{(1)}_4\\&+G^{(3)}_{1,2,4}G^{(1)}_3+G^{(3)}_{1,3,4}G^{(1)}_2+G^{(3)}_{2,3,4}G^{(1)}_1)\\&+2\rho_1^2\rho_2^2(G^{(2)}_{1,2}G^{(2)}_{3,4}+G^{(2)}_{1,3}G^{(2)}_{2,4}+G^{(2)}_{1,4}G^{(2)}_{2,3}).\end{aligned}\tag{8}$$

In this way the *k*-electron correlation function for partially polarized electrons is expressed in terms of the one-electron, two-electron, etc. up to *k*-electron correlation functions for polarized electrons, and the degree of polarization [that is connected with ρ_1 and ρ_2 via relations (2)].

The considerations made in Secs. III–V, as well as results (3) and (6), can also be applied step by step to a chaotic photon field, because up to now we have not supposed anything about the quantum statistics of the particles. In fact, these statistics are hidden in the spin-polarized correlation functions *G*, and in this way are also reflected in the correlation function $O^{(k)}_{1,\ldots,k}$. As mentioned, the similarity between electrons and photons in this sense comes from the same dimensions of the photon polarization Hilbert space and the electron spin Hilbert space.

We return to the electrons again. According to Refs. [4,6], the *k*-electron correlation function for a spin-polarized chaotic electron beam can be expressed as

$$G^{(k)}_{1,\ldots,k}=G^{(1)}_1G^{(1)}_2\cdots G^{(1)}_k\det\hat{\gamma},\tag{9}$$

where $\hat{\gamma}=(\gamma_{ij})$ is a matrix composed of the complex degrees of coherence γ_{ij} at the points $(\vec{r}_i,t_i)$ and $(\vec{r}_j,t_j)$. Combining Eqs. (6) and (9), we arrive at an explicit form for the correlation function for a chaotic electron beam with an arbitrary spin polarization.

VI. INFLUENCE OF POLARIZATION ON MULTIELECTRON CORRELATIONS

To see how the spin polarization influences the correlations in an electron beam, we first return to the case when $k=2$. According to Eq. (9), the two-electron correlation function for a spin-polarized chaotic electron beam is equal to

$$G^{(2)}_{1,2}=G^{(1)}_1G^{(1)}_2(1-|\gamma_{12}|^2),\tag{10}$$

where we used the fact that $\gamma_{11}=\gamma_{22}=1$. With the help of Eqs. (2) and (3), for $O^{(2)}_{1,2}$ we then obtain

$$O^{(2)}_{1,2}=G^{(1)}_1G^{(1)}_2\left(1-\frac{1+P^2}{2}|\gamma_{12}|^2\right).\tag{11}$$

If there were no correlation between the detection probabilities at the points $(\vec{r}_1,t_1)$ and $(\vec{r}_2,t_2)$, the correlation function $O^{(2)}_{1,2}$ would simply be equal to the product of the one-electron correlation functions $O^{(1)}_1=G^{(1)}_1$ and $O^{(1)}_2=G^{(1)}_2$. Therefore, the second term in parentheses in Eq. (11) is responsible for the two-electron correlation. As we can see, this term increases with the increasing degree of polarization of the beam, varying between one half for an unpolarized beam and unity for a completely polarized beam. For electrons, this result has been known [1], but it has been derived heuristically only until now. For photons a similar effect of the polarization on the correlation function is known [9].

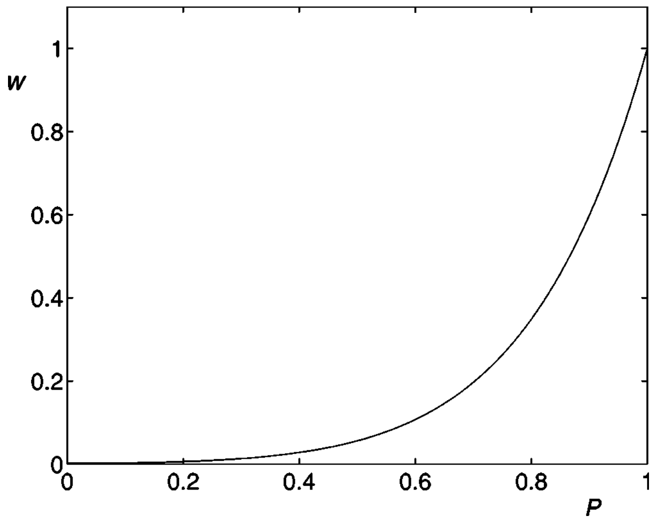

FIG. 1. The factor $w=\rho_1^k+\rho_2^k=[(1+P)/2]^k+[(1-P)/2]^k$ expressing the intensity of the *k*-electron correlation as a function of the degree of polarization *P* for $k=10$. As the figure shows, if *w* is comparable to unity, thus making the *k*-electron correlations observable, one needs a beam of a relatively high degree of polarization.

Next we go over to the case of an arbitrary *k*. According to Eq. (6), the factor $w=(\rho_1^k+\rho_2^k)$ expresses the weight of the *k*-electron correlation in the partially polarized beam compared to a completely polarized beam, because the *k*-electron correlation is given just by the correlation function $G^{(k)}_{1,2,\ldots,k}$. It is evident that for large *k* the factor *w* becomes small as soon as *P* differs even slightly from unity. Figure 1 shows the dependence $w(P)$ for $k=10$. For example if $P=0.7$, then *w* is equal to only about 0.2, so the ten-electron correlation is reduced to one fifth with respect to a polarized beam. Thus we must conclude that if no beam with a high degree of polarization is available, it is difficult to observe correlations of higher orders. On the other hand, from the experimental point of view, observing even two-electron correlations is very difficult [2,10]. In comparison to the extreme difficulty of, say, a ten-electron correlation experiment, making a 99% polarized electron beam seems to be an easy task, and in this way the spin degree of freedom should have no limitation effect on the measurement of multielectron correlations.

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